**SEPARATION OF CHARGED PARTICLES TO THEIR INITIAL MAGNETIC FIELD LINES IN THE SIMPLE TURBULENT MAGNETIC FIELD MODEL**

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## Abstract

In general, when charged particles move in the magnetic field, they move around the magnetic field as spiral orbits due to Lorentz force. If the magnetic field is turbulent such as that in the interplanetary space, the charged particles and their initial magnetic field line are separated. In this work, we are interested in the behavior of charged particles transiting along their initial magnetic field lines in turbulent magnetic field as in the interplanetary space in our computer simulations. We use the two-component (2D+slab) model for the magnetic field. Here we use a simple model for the turbulent magnetic field in order to understand the behavior of separation. We specify the potential function as $a(x, y)$ for 2D field as Gaussian function and the power spectrum of slab turbulence as Kolmogorov spectrum. In the simulations, the 3000 charged particles are released randomly at the initial pitch angles of 0-90 degrees. We solved Newton-Lorentz force and field line equation to find trajectory of the charged particles and magnetic field lines by using Runge-Kutta method with adaptive time stepping regulated by the fifth-order error estimation. Then we compute the positions of guiding centers of these charged particles trajectories and then trace their corresponding field lines which initially started at the initial guiding centers of the charged particles. After that we analyzed the relationship between them by calculating the mean squared perpendicular displacement of the separation and time. From our results we found that the separation depends on locations where they started and the structure of the magnetic field they experienced. The particles followed the field lines at the beginning and they drifted and left the 2D flux tube. Finally, they undergo subdiffusive due to the slab turbulence at the later time.

**Keywords:** charged particles, magnetic field, Newton-Lorentz force, separation, turbulence.

## Introduction

The Sun consists of plasma – an ionized gas. It is fully composed by charged particles such as proton, electron, hydrogen and helium ions. Some of these charged particles continuously flow out from the Sun into the interplanetary space and drag the magnetic field into the space. The plasma that flow out from the Sun is called as solar wind. Since the solar wind is turbulent, it causes the dragged interplanetary magnetic field is turbulent as well. Moreover, some of high energetic charged particles called solar energetic particles (SEPs) are released from the Sun. These particles transport along the interplanetary magnetic field. Many scientists are interested in the transport of these high energetic charged particles in the magnetic field in interplanetary space because some of them effect to the Earth and we can also learn more about the physics of the charged particles in magnetic turbulence. There are many models used to explain behavior of charged particles. It can be seen that some phenomena are explained by modeling the structure of magnetic field in interplanetary space. One of several models is two component model or 2D+slab model. It was developed by the observation solar wind fluctuations are concentrated at nearly parallel and perpendicular wave number (Matthaeus et al. 1990). This model was successful to explain the transport of SEPs in the interplanetary space. For example, the “dropout” phenomena are described by conditional statistics that magnetic field trajectories are trapped when they initially start near O-points (inside the 2D island) and rapidly spread when they locate near X-points (Ruffolo et al. 2003). The model is also used to study the transport of charged particles and magnetic field lines. The strong two-dimensional field can inhibit the random walk of field lines due to a slab field component (Chuychai et al. 2005). In addition, the two component model of magnetic field causes the temporary topological trapping and escape of charged particles in a flux tube as a cause of delay in time asymptotic transport. The result tells us that the charged particles have lower energy and will be deeply embedded in stronger flux tubes (Tooprakai et al. 2007). We can see that the charged particles in the space are influenced by the turbulent magnetic field. Therefore, it is also interesting to see how the charged particles correspond with their initial field lines in term of the separation. When the dimension of 2D+slab magnetic field model is reduced, it has effect to separation of the charged particles from their field line and no cross-field diffusion (Chuychai et al. 2011). From the previous work, we studied about the effect of the initial pitch angles of charged particles to the separation on the turbulent magnetic field lines. It showed that initial pitch angles as 0-30 degrees have more separation than other ranges of the pitch angles (Wikee et al. 2012).

In this work, we use a simple turbulent magnetic field model as 2D Gaussian magnetic field + slab turbulence (Chuychai et al. 2005). We model the 2D potential function as a Gaussian function to study the separation of the charged particles from their initial field lines when we start the particles from various distances from the center of the 2D flux tube. Perhaps this work can help us better understand the mechanism of the separation as well as the drift motion of charged particles from the magnetic field lines in space.

## Methodology

Magnetic Field Model

The magnetic field model consists of 2D+slab field or two component model as we can see in Figure 1. The magnetic field in 2D+slab model which is composed of the mean field and fluctuations can be written as

$$\vec{B} = \vec{B}_0 + \vec{b}(x, y, z)\,, \tag{1}$$

where $\vec{B}_0$ is a constant mean field in $z$ direction and $\vec{b}$ is the transverse fluctuation perpendicular to the mean field. Then we can write the transverse fluctuation as

$$\vec{b}(x, y, z) = \vec{b}^{2D}(x, y) + \vec{b}^{slab}(z)\,. \tag{2}$$

For the 2D part, we use Gaussian 2D flux tube (Chuychai et al. 2005) in this work. That is the potential function $a(x, y)$ is represented with a Gaussian function:

$$a(r) = A_0 \exp[-\frac{r^2}{2\sigma^2}], \tag{3}$$

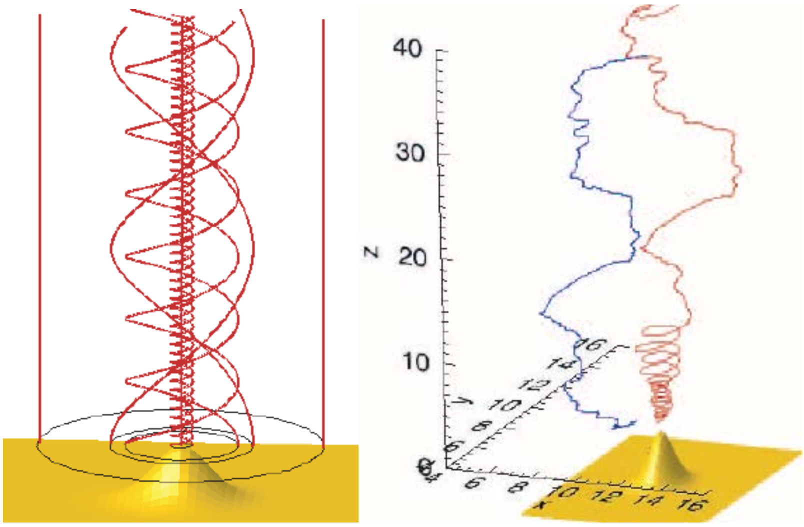
where $A_0$ is the central maximum value, $\sigma$ determines the width of the Gaussian, and the distance $r$ is measured from the center of the Gaussian flux tube. From $\vec{B} = \vec{\nabla} \times \vec{A}$, where $\vec{A}$ is vector potential, it can be written as

$$b^{2D}(r) = \frac{ra(r)}{\sigma^2}\hat{\theta}. \tag{4}$$

For the slab field, it will be the fluctuation in $z$ direction, we use characteristic of turbulence to model the magnetic field which the shape of power spectrum in inertial range obeys Kolmogorov's 5/3 law. Then we specify power spectrum for slab part as

$$P_{xx}^{slab}(k_z) = \frac{C}{\left[1+(k_z\lambda)^2\right]^{5/6}}, \tag{5}$$

where $C$ is constant, $k_z$ is magnitude of the wave vector, and $\lambda$ is a coherence length. For the slab turbulence, after we specify the power spectrum and generate the magnetic field in $k$ space. We use invert fast Fourier transform to convert the magnetic field back into real space.




(a) (b)

**Figure1** showing (a) the example of magnetic field lines in a single 2D Gaussian magnetic field and (b) the example of two magnetic field lines in a single 2D Gaussian field plus slab turbulence.

Equation of Motion

The trajectories of charged particles are numerically computed by solving the modified Newton Lorentz-force equation

$$\frac{dv'}{dt'} = \alpha(v' \times B'), \tag{6}$$

where $\alpha = (qB_0\tau_0)/(\gamma m_0)$, $v', B'$, and $t'$ are normalized quantities which have the units scaled to the speed of light ($c$), the mean magnetic field ($B_0$), the time scale $\tau_0 = \lambda / c$, $q$ and $m_0$ are charge and mass of the particles, respectively . Equation (6) is solved by using Runge-Kutta method with adaptive time stepping regulated by a fifth-order error estimate step (Dalena et al. 2012). The solutions of this equation are the positions and velocities of the charged particles. The angles between the direction field $\vec{B}$ and the direction of the charged particles called the pitch angles $(\theta)$ are also computed. Here we can also find the pitch angle by

$$\theta = \cos^{-1} \frac{\vec{v}.\vec{B}}{|\vec{v}||\vec{B}|}. \tag{7}$$

After we have the charged particle trajectories, we find radius of curvature $(\vec{\rho})$ of the charged particles from

$$\vec{\rho} = \frac{\vec{B} \times \vec{p}}{qB^2}, \tag{8}$$

where $\vec{p} = \gamma m_0 \vec{v}$ is the particle momentum. Then we can compute the guiding centers from the radius of curvature of the charged particle orbits from $\vec{r}_{GC} = \vec{r} - \vec{\rho}$ (see also in Figure 2).

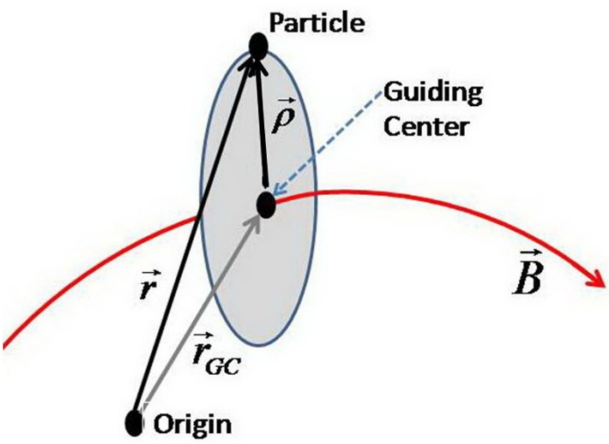



**Figure2** Showing radius of curvature, particle position and guiding center position.

Field Line Equation

When we know initial position of guiding centers of charged particles, we bring the initial guiding center of each charged particle to be the initial position of the magnetic field line. After that we trace the magnetic field line that is tangent everywhere to the magnetic field. The equation for magnetic field line is

$$\frac{dx}{dz} = \frac{b_x}{B_0} \text{ and } \frac{dy}{dz} = \frac{b_y}{B_0}. \tag{9}$$

The equation (9) is also solved by using Runge-Kutta method with adaptive time stepping regulated by a fifth-order error estimate step as same as we solve trajectories of charged particles (Dalena et al. 2012). The outputs from solving equation (9) are the positions of the magnetic field lines.

In data analysis, the mean squared displacement are calculated from position of guiding center of charged particles $(x_{GC})$ and their corresponding magnetic field line $(x_{FL})$ at the same z-coordinate as a function of time the equation is

$$\left\langle \left( x_{GC}(t) - x_{FL}[z(t)] \right)^2 \right\rangle, \tag{10}$$

where $z(t)$ is the *z*-coordinate of the particle guiding center at time *t*. Note that $x_{FL}$ is single valued because we assume transverse fluctuations, so $B_z = B_0$ constant and the magnetic field cannot back track in *z* coordinate of the particle guiding center at regular time intervals.

Numerical Simulation Setup

The slab magnetic field is generated on grid points in the simulation box, and we set the box length in $x, y$ direction as $100\lambda$, $z$ direction as $100{,}000\lambda$ while 2D magnetic field is computed from the function in equation (4). The number of grid points is $N_z = 4{,}194{,}304$. Note that the grid sizes in $x$ and $y$ are not necessary here because the 2D magnetic field is generated from the function. In the simulations, we set $B_0$ =0.5 nT, $\lambda = 0.02$ AU. For Gaussian function, we give the width of the Gaussian $\sigma$ as $0.5\lambda$. We define $b_{2D}^{\max} / B_0 = 1.0$ and $\left(b_{2D}^{\max} / \delta b_{slab}\right)^2 = 20$ that means the 2D flux tube is very strong compared with slab turbulence. The test particles are designed to represent protons that have energy 100Mev. In our simulations, all units of lengths are scaled with $\lambda$ and the unit of the time is scaled by $\lambda/c$.

## Results

The 3000 charged particles are released at random pitch angles from 0 to 90 degrees and on various distances from the center of the 2D Gaussian island ($r_0$) as $0.1\lambda$, $0.3\lambda$, $0.5\lambda$, $0.7\lambda$ and $0.9\lambda$. The simulation results show that the separation behaviors between the charged particles and the corresponding field lines can be divided into 4 regimes as we can see in Figure 3. We can explain the mechanism of separation in each regime by relating to the structure of the 2D Gaussian and slab turbulent magnetic field.

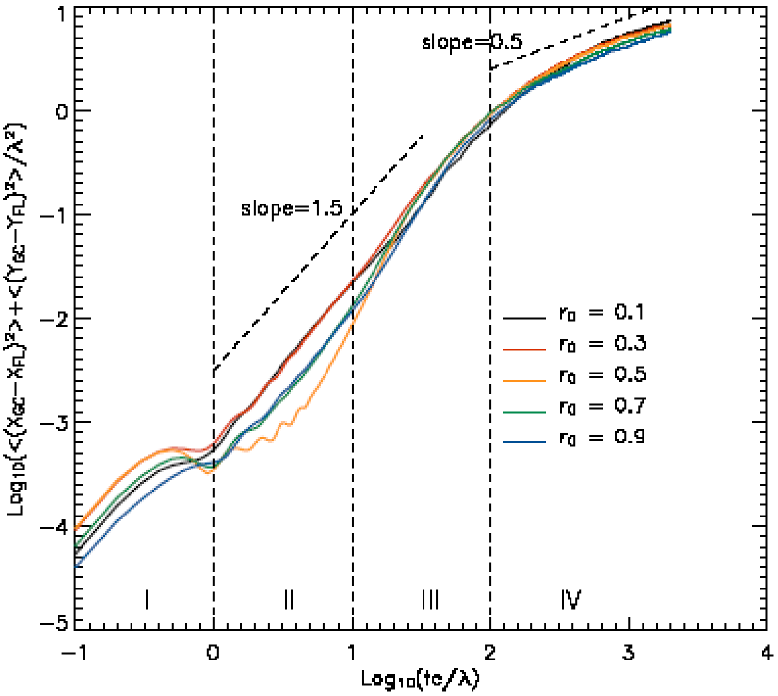



**Figure3** The results of the separation of charged particles and their corresponding field lines in the log-log scale.

I. At the initial times, when $tc/\lambda < 1$

We found that the separation of the charged particles in initial time which are starting at the radius of $0.3\lambda$ and $0.5\lambda$ are higher than $0.1\lambda$, $0.7\lambda$ and $0.9\lambda$. It seems that the separation of charged particles depend on the intensity of 2D magnetic field because at the positions of $0.3\lambda$ and $0.5\lambda$ from center of Gaussian function have the higher intensities of magnetic field than the radii as $0.1\lambda$, $0.7\lambda$ and $0.9\lambda$. The profile of the intensity of 2D Gaussian flux can be seen in Figure 4. The maximum of 2D magnetic field is at the width of Gaussian function ($\sigma$) and the decrease when the radius towards to the center as well as when they go outside.

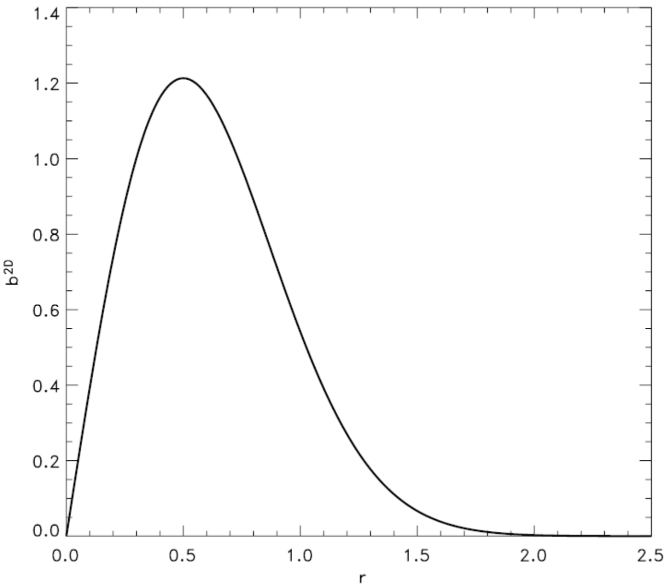



**Figure 4** The profile of the 2D Gaussian magnetic field along the distance from the center of the flux tube.

II. At the early times, when $tc/\lambda \sim 1-10$

In this regime, the charged particles follow their corresponding field lines for a while and start to escape from the influence of the 2D flux tube. There are interesting features in this regime. The particles start at $r_0 = 0.5\lambda$, at the strongest 2D magnetic field, have lower separation than the others as shown in Figure 3. The particles are strongly tied with their initial field lines and slowly drift out from the field lines. The behaviors of the particles that start inside and outside 2D islands are clearly distinct here no matter where they initially locate. As we can see from Figure 1, the particles start inside 2D is land at $r_0 =$ $0.1\lambda$ and $0.3\lambda$ have about the same separation. We can see also from the ones starting outside at $r_0 =$ $0.7\lambda$ and $0.9\lambda$. This is corresponding with the effect of sharp gradient from the 2D magnetic field from the previous work (Chuychai et al. 2007;Tooprakai et al. 2007).

III. At intermediate range, when $tc/\lambda \sim 100$

In the intermediate range, we found that the charged particles released near high intensity of 2D magnetic field separate more than other initial positions. As presented in Figure 5, the charged particles released at radii as $0.3\lambda$, $0.5\lambda$ and $0.7\lambda$ separate more than the particles released at the radii as $0.1\lambda$ and $0.9\lambda$. Therefore, the separation of the charged particles to their corresponding magnetic field lines is also related to intensity of 2D magnetic field at any radius in this range.

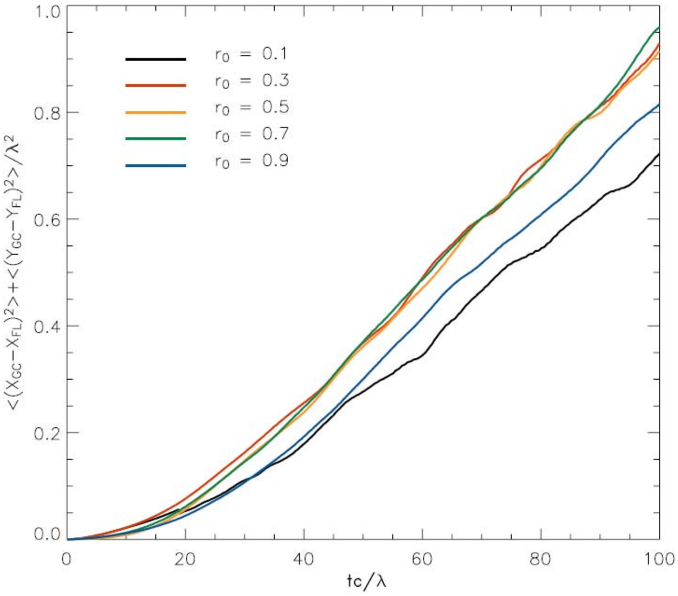



**Figure 5** The mean squared perpendicular displacement and time in intermediate range.

IV. At final time, when $tc/\lambda >> 100$

From the final range in Figure 6, we can see that the charged particles are released at radius as $0.1\lambda$ separate faster than the other radii. It seems the separation is related to the radius of releasing the charged particles. If the charged particles are released near the center of Gaussian function, they separate from their initial field lines more than the other positions. In this range, the transition of the charged particles and their corresponding magnetic field lines are uncorrelated. Note that the corresponding length scale of the uncorrelation between particles and field lines is in the order of coherence length scale ($\lambda$). The charged particles are mainly influenced by slab turbulence and undergo subdiffusive as we can see from the slope = 0.5 in Figure 3. We normally find subdiffusive process when charged particles transport in pure slab magnetic field (Tooprakai et al. 2007). It occurs because the charged particles are scattering and moving back and forth due to the slab turbulence.

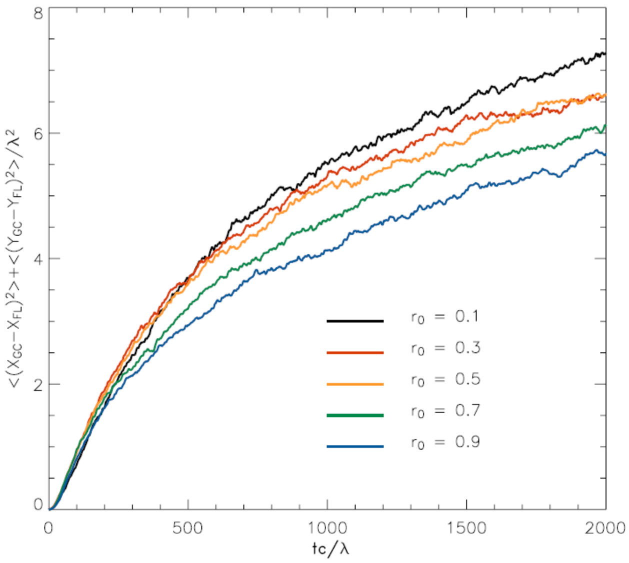



**Figure6** The mean squared perpendicular displacement and time in final range.

**Conclusions**

From the results, the separation of the charged particles relate with the distance from the center of the Gaussian flux tube $(r_0)$ and where they experience the structure of the magnetic field. When the charged particles are released at high intensity of 2D Gaussian field, the separation is lower than the others at the early times. They try to follow the field line and slowly drift to outside the 2D flux tube. The sharp gradients of 2D field can distinct behavior of the particles inside and outside the island in this regime. It corresponds with the suppressed diffusive regime in the previous work (Chuychai et al. 2007;Tooprakai et al. 2007). After that they separate more than other positions, we can see in intermediate time. In addition, for final time the separation of the charged particles is uncorrected with the starting point to release the charged particles. The separation of the charged particles depends on distance from the center of the Gaussian function and become subdiffusive. Finally, this work can help us to understand more about the relation of the separation between guiding centers of charged particles and magnetic field lines. In the future work, we will study more about theory and simulations in order to describe the mechanism or characteristic of the separation including the effect of pitch angle between charged particles and magnetic field lines.

**Acknowledgements**

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